NOTES

A Method of Estimating and Minimizing the Error of Measurement of the Rate of a Radioactive Exchange Reaction

By Norman Davidson and John H. Sullivan

It has been explicitly pointed out by several authors that for a radio-active exchange reaction between two components in chemical equilibrium, the activity of either component varies in a simple exponential manner with time.[1,2] We present here, as a consequence of this analysis, a method for: (a) the estimation of the error, due to the error of radioassay, in the rate constant for the exchange reaction, as a function of the extent of exchange; (b) the selection of the optimum degree of exchange to minimize this error.[3]

For the case where the rate is measured by the decrease in the activity of the component that initially contained all the activity, the integrated rate expression is

$$\ln\left[\left(\frac{a}{b}+1\right)\frac{x}{c}-\frac{a}{b}\right] = -R(a,b)\,\frac{a+b}{ab}\,t \qquad (1)$$

In this relation: *a* is concentration of component A that initially contained all the activity; *b* is concentration of initially inactive component B; *c* is initial activity of A; *x* is activity of A at time *t* and $R(a, b)$ is the rate of exchange.

For very short times of reaction *x* will be almost the same as *c* and the error in the rate large. For long times of reaction, the components will be almost in equilibrium with respect to the distribution of activity which will then change but little with time. It is often the case that the errors in *a*, *b*, *t* are small compared to the errors of radioassay in *x* and *c*. Set $z = x/c$ and $s = R((a + b)/ab)t$. The variable *s* is dimensionless and proportional to two factors: (1) *Rt*, the total number of concentration units (*i. e.*, atoms/cc. or moles/liter) that have undergone mutual exchange, and (2) the term $(a + b)/ab$. (Displayed in the form $s = Rt/b + Rt/a$, it is evident that *s* is the sum of the number of exchanges per atom of *a*, and the number of exchanges per atom of *b*). Then

$$-\frac{\mathrm{d}\ln s}{\mathrm{d}\ln z} = \frac{1+(a/b)\exp(s)}{s} = E(a/b,s) \qquad (2)$$

The relation $\sigma_s = |\mathrm{d}s/\mathrm{d}z|\sigma_z$ holds for the standard deviations, σ_s and σ_z, of *s* and *z*, because of the assumption that the error in *s* is due entirely to the error in *z*.[4] Then

$$\frac{\sigma_s}{s} \times \frac{z}{\sigma_z} = -\frac{\mathrm{d}\ln s}{\mathrm{d}\ln z} = E(a/b,s) \qquad (3)$$

For $\sigma_z, (\sigma_z/z)^2 = (\sigma_x/x)^2 + (\sigma_c/c)^2$; the standard deviations of the activities may be due to statistical counting errors or may be manipulative errors determined by reproducibility tests. It is often the case that the fractional error in $x, (\sigma_x/x)$, is a constant; this is roughly true, for example, if the error is principally a counting error and one counts all samples to the same number of counts, or if the error is a manipulative error in preparing chemically identical samples for radioassay. For such cases, the minimum of *E* locates the point for minimum error in *s* (and hence in the rate function *R*). Knowing the value of *s* corresponding to the minimum value of *E* and using a preliminary value of *R*, the optimum *t* may be chosen.

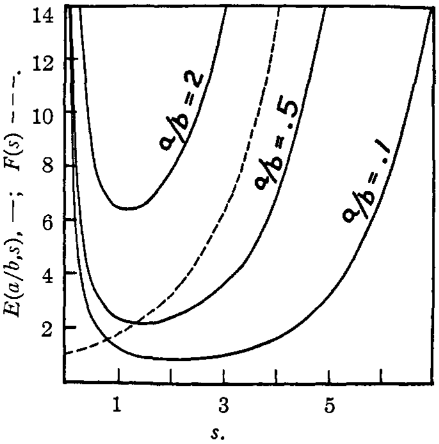

Fig. 1.—Error functions for exchange reactions.

The minimum of the error function, *E*, may be found by solution of the equation: $s - 1 = (b/a)\exp(-s)$. It will generally be more useful to construct a family of curves like those in the figure so that the error in *s* will be known for any *s* (or *z*). Estimates of the standard deviation in *s* obtained in this way are useful, for example as weighting factors in averaging data or in least squares treatments of data on the variation of rate of exchange with temperature or ionic strength. For such applications, the function *E* is useful even when the fractional error of radioassay is not constant. For these cases, knowing σ_x as a function of *x* and hence (for given *a* and *b*) as a function of *s*, one could construct the function, $E(s)\sigma_x(s)/x(s)$, and thus select the point of minimum error. For the usual case where σ_x/x does not change too rapidly with *x*, it may be sufficient to select by inspection a point in the region of the minimum of the *E*(*s*) curve without making the more elaborate calculations required to select the optimum point.

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(1) McKay, *Nature*, **142**, 997 (1938).

(2) Duffield and Calvin, This Journal, **68**, 557 (1946).

(3) Roseveare, *ibid.*, **53**, 1651 (1931), has applied similar arguments to the problem of estimating and minimizing the error in the rate constants for chemical reactions.

(4) See, for example, Margenau and Murphy, "The Mathematics of Physics and Chemistry," D. Van Nostrand Co., New York, N. Y., 1943. p. 498.

For the case where the rate of exchange is measured by the growth of activity, y, in B

$$y(t) = y(\infty) \quad [1 - \exp(-s)] \qquad (4)$$

$$d \ln s / d \ln (y/y(\infty)) = (\exp(s) - 1)/s = F(s) \qquad (5)$$

The figure contains a plot of this function, too. In this case, of course, the fractional error in y cannot possibly be constant as y approaches zero; an estimate of σ_y as a function of y is needed for selecting an optimum reaction time or calculating the error of a particular determination.

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GATES AND CRELLIN LABORATORIES OF CHEMISTRY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA 4, CALIFORNIA
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A Convenient Synthesis of N,N-Dimethyl-*p*-nitroaniline and N,N-Dimethyl-*o*-nitroaniline

BY TOD W. CAMPBELL

There are a number of methods in the literature for the preparation of N,N-dimethylnitroaniline[1]; but most of them are troublesome and result in low yields of a product which is usually of not too high purity. The methods in the literature[1g,i] which give acceptable or good yields involve a reaction in which a nitrohalobenzene is allowed to react with dimethylamine.

The author has found that this reaction is most conveniently brought about by refluxing a pyridine solution of nitrohalobenzene with a mixture of dimethylamine hydrochloride and sodium bicarbonate. The desired product is obtained in virtually quantitative yield; in the case of the para isomer, the product can be crystallized directly from the reaction solvent in a high state of purity. This procedure is therefore recommended for the preparation of these two substances.

Experimental Part

***p*-Nitrodimethylaniline.**—A mixture of 42 g. of *p*-bromonitrobenzene, 300 cc. of pyridine, and 50 g. of sodium bicarbonate was placed in a 500-cc. round-bottom flask. To this mixture was added 30 g. of dimethylamine hydrochloride dissolved in about 10 cc. of warm water. The mixture was refluxed for ten hours. Mechanical stirring was not employed, since serious bumping did not occur. At the end of the reflux period, the hot solution was filtered free of inorganic salts, and the latter was extracted with 200 cc. of acetone, which was added to the pyridine solution. The mixed extracts were boiled, and water added to near the cloud point. On cooling, bright yellow needles of *p*-nitrodimethylaniline, 1–3 cm. in length, crystallized out. The melting point was observed to be 163.7–164.1° (lit. 163–166°) on a calibrated Anschütz thermometer in a Hershberg apparatus. The mother liquor on concentration to one third of its original volume gave an additional small yield of fine yellow needles, which had a melting point anywhere from 1–10 degrees low, for various experiments. One recrystallization from methanol raised the melting point to 163.5–164°. The over-all yield of pure product was 32.4–33.6 g. (94–97%).

***o*-Nitrodimethylaniline.**—The above procedure was employed to prepare the ortho substituted derivative. Ten grams of *o*-nitrochlorobenzene gave 8.9 g. (85%) of product; b. p. 149 at 20 mm.; $n^{25}D$ 1.6080.

Anal. Calcd. for $C_8H_{10}N_2O_2$; C, 57.81; H, 6.06. Found: C, 57.53; H, 6.21.

UNIVERSITY OF CALIFORNIA
LOS ANGELES 24, CALIF.
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[1] (a) Beilstein "Handbuch," Vol. XII, 690, 714, and first Supplement; (b) Le Fevre, *J. Chem. Soc.*, 147 (1930); (c) Davies, *Bull. soc. chim.*, [5] **2**, 295 (1935); (d) Donald and Reade, *J. Chem. Soc.*, 53 (1935); (e) Marsden and Sutton, *ibid.*, 599 (1936); (f) Shorygin, Topchier and Anan'ina, *J. Gen. Chem.* (*U. S. S. R.*), **8**, 981 (1938); (g) Hodgson and Kershaw, *J. Chem. Soc.*, 280 (1930); (h) Evans and Williams, *ibid.*, 1199 (1939); (i) Senear, Rapport, Mead, Maynard and Koepfli, *J. Org. Chem.*, **11**, 378 (1946).

Vapor Density of Diborane[1]

BY E. M. CARR, J. T. CLARKE AND H. L. JOHNSTON

In connection with the process of adding diborane to a calorimeter it was necessary to determine its density at 275.16°K. The diborane was obtained from the Naval Research Laboratory, subjected to a two-plate distillation and shown by the cryoscopic method to have a purity of 99.95 mole per cent. The diborane was introduced into an evacuated, weighed and calibrated 1-liter Pyrex bulb immersed in a constant temperature waterbath at 275.16°K. and constant to 0.01° C. during the measurement. The pressure was read on a 15 mm. i. d. manometer using a Gaertner cathetometer and a standard meter bar in an insulated case; the readings were converted to standard conditions and meniscus corrections made according to Cawood and Patterson.[2] The bulb was then reweighed (using a similar bulb as tare) on a Troemner 4-kg. balance. One bulb had 1/8 inch Pyrex helices with a surface area 16.0 times that of the interior part of the bulb added to it. Since all density measurements were made at approximately atmospheric pressure the amount of adsorption was assumed to be constant.

The results were used to calculate the density and the second virial coefficient B in the equation

$$n - A = \frac{PV}{RT + PV}$$

where A equals moles of diborane adsorbed on the surface of a 1-liter bulb and was found to have a value of 2.4×10^{-5} mole. A summary of the data is

Pressure, atm., P	Temperature, °K., T	Moles in gas phase, $n - A$	Volume of bulb, ml., V	Second virial coef. B (ml.) 275.16° K.	Density g./l. 1 atm. 275.16°K.
0.98832	275.20	0.048232	1090.2	−247	1.2398
.98201	275.14	.047296	1076.4	−233	1.2393
.99445	275.16	.048488	1090.2	−223	1.2386
.94553	275.16	.046104	1090.2	−234	1.2393
.96219	275.17	.046314	1076.4	−227	1.2388
.97098	275.15	.046690	1076.4	−199	1.2374
		Average		−227	1.2389
		Av. deviation		±11	±0.0006

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[1] This work was carried out under contract between the Office of Naval Research and The Ohio State University Research Foundation.

[2] Cawood and Patterson, *Trans. Faraday Soc.*, **29**, 514–523 (1933).